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Theoretical Investigations on Dimensional Analysis of Ball Bearing Parameters by using Buckingham Pi-Theorem

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Abstract

Dimensional analysis is a very powerful and general tool for use in analysing and understanding problems in engineering and in particular, in mechanics and transport phenomena. The main objective of the present work is to study the characteristics of various bearing parameters under low, medium and high temperatures conditions. Dimensional analysis is useful computing dimensionless parameters and provides answer to what group of parameters that affecting the problem. This Dimensional analysis can be accomplished by using Buckingham π-theorem. Dimensional analysis leads to a reduction of the number of independent parameters involved in a problem. These independent parameters get expressed as dimensionless groups. These dimensionless groups are always ratios of important physical quantities involved in the problem of interest. In modelling and experiment, its main function is to reduce the amount of independent variables, to simplify the solution and to generalize the results thereof. It can become an effective method, especially if a complete mathematical model of the investigated process is not known. The present work also attempts the application of Buckingham pi-theorem to find what parameters are influencing the bearing system and using dimensionless parameters the characteristics are studied.

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Keywords: Dimensional analysis,Buckingham pi-theorem.

1. Introduction

Rotating shafts are employed in industrial machines such as steam and gas turbines, turbo generators, internal combustion engines, reciprocating and centrifugal compressors, for power transmission. On account of the ever increasing demand for power and high speed transportation, rotors of these machines are made extremely flexible,

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which makes the study of vibratory motion an essential part of design. The shafting of these machine installations is subjected to torsional and bending vibration and in some cases unstable condition of operation.

Dimensional analysis involves a dimensional model analysis of acting quantities in the investigated process. It enables one to determine, in a simple algebraic way, dimensionless similarity criteria and functional relations, represented amongst them by a criterion equation. Further, it enables the conversion of physical quantities into other various fundamental sets of measuring units, the conversion of measuring units and other procedures. In modelling and experiment, its main function is to reduce the amount of independent variables, to simplify the solution and to generalize the results thereof. It can become an effective method, especially if a complete mathematical model of the investigated process is not known.

This is a method, simple from the practical point of view, which does not enable either solving a problem completely or revealing important inner couplings of an investigated phenomenon. However, it is an extraordinarily effective means of obtaining an idea about the behaviour of a phenomenon if neither its complete mathematical nor physical descriptions are known. Usually, it is an important physical tool in every more complicated physical, scientific or industrial experiment. The main functions of dimensional analysis are the following.

- Determination of the number and form of dimensionless quantities which represent the similarity criteria.
- Reduction of the numbered independent variables in an experiment, simplification of the solution and generalization of its results.
- Conversion of the basic set of units of the measurement,
- Conversion of physical quantities into another basic set of units of measurement,
- Determination of functional relations in cases where the solver does not know more detailed information of the physical principle of the investigated phenomenon and no complete mathematical description of the phenomenon is known.

In application of dimensional analysis, the highest efficiency is reached in its combination with general physical ideas obtained by a solver directly from experiments. The depth of previous knowledge of the physical principles of the investigated phenomenon can influence and extend considerably the possibilities of the dimensional analysis

Nomenclature

Fr	Reaction Forces	N	Load carrying Capacity
g	Acceleration due to Gravity	F_b	Frequency of the bearing
n	Shaft Speed	μ	Dynamic viscosity
P	Power Supply	D_{ball}	Bearing Size
q	Heat Generated in the Bearing	C_{ball}	Clearance of Ball Bearing
T_a	Surface Temperature of the Bearing	k	Thermal Conductivity
h	Convective heat transfer coefficient	ρ	Shaft density
EJ	Flexural Rigidity	I	Moment of Inertia
S	Stiffness	k_{ref}	Thermalconductivity
$D_{ball\ ref}$	Bearing Size	μ_{ref}	Dynamic viscosity
n_{ref}	Shaft speed	$T_{initial}$	Temperature initial
N_{ref}	Load carrying Capacity	Fb_{ref}	Frequency of the bearing

2. Literature Review

The idea of experimentation with a different, rather than the actual, dimension was suggested by several individuals independently. Some attribute it to Newton, whocoined the phrase of “great Principle of Similitude.” Later, Maxwell a Scottish Physicist played a major role in establishing the basic units of mass, length, and time as building blocks of all other units. Another example, John Smeaton[1] was an English civil and mechanical engineer who study relation between propeller/wind mill and similar devices to the pressure and velocity of the driving forces.

Jean B. J. Fourier [1] first attempted to formulate the dimensional analysis theory. This idea was extend by William Froude by relating the modelling of open channel flow and actual body but more importantly the relationship between drag of models to actual ships. While the majority of the contributions were done by thermo–fluid guys the concept of the equivalent or similar propagated to other fields.AiméemVaschy, a German Mathematical Physicist suggested using similarity in electrical engineering and suggested the Norton circuit equivalence theorems.Rayleigh probablywas the first one who used dimensional analysis to obtain the relationships between the physical quantities.Osborne Reynolds [2] was the first to derive and use dimensionless parameters to analyse experimental data.

Buckingham [1]culminated the dimensional analysis and similitude and presented it in a more systematic form. In the about the same time German engineer, developed the dimensional analysis.Bruwellobtained the shaft orbits for various modes of dynamic loading of long bearing using Sommerfeld conditions. Prior to the advent of computers, Cameron[3] and Wood used Southwell's Relaxation method to solve the Reynolds Equation for finite full journal bearings ranging from $L/D = \infty$ to ¼.

Wilcock [4]in a series of experiments with 8-in. journal bearings discovered that their performance was seriously altered when operated in the turbulent regime.Petrusevich[5] obtained the solutions for film thickness in gears which included the elasticity equations and, in the process, discovered the essential and typical shape of elastohydrodynamic pressure profiles.

Ocvirk[6,7]provided a detailed and full solution to the problem of short bearings. It is a most simple, compact, and elegant solution which for analytical manipulations is without peer. And despite its label of infinitely short, it is pretty much valid to L/D ratios of up to ½, which is the design range of most modern bearings. Sommerfeldand Walter [8]used a Gaussian algorithm for solving finite difference equations for both 360^0 and 180^0 arcs.

The first use of modern computers in the solution of the finite Reynolds equation using the proper boundary conditions was made by Pinkus[9] He obtained the solutions not only for circular but also for elliptical and three-lobe bearings for L/D ratios ranging from 1.5 to 0.25, as well as for finite bearings for various arcs and (R_2 /R_1) ratios.Bruwellused the same approach as Swift, to obtain the solutions for the dynamically loaded bearings based on the short bearing theory.Raimond and Boyd[10] provided the solution for full and partial journal bearings for L/D ratios of 0.25, 0.5, and 1 for incompressible fluids and also presented the results for gas bearings for L/D ratios of 0.5, 1, 2.Gross assembled a tabulation of finite gas bearing solutions for various operating conditions.

Booker[11] first presented the "mobility" method, a new concept of treating dynamically loaded bearings.Ng and Pan[12] using eddy viscosity derived the bearing performance equation for the turbulent flow conditions.Lund [13]introduced the interaction of stiff and flexible rotors with the bearings in determining stability.The Reynolds equation in its finite form and with the correct boundary conditions was solved for nearly any bearing configuration for both liquid and gas lubricants. Gears, Rolling Element Bearings, and traction drives received a workable and solid theory to calculate performance. Bearings linked to rotor dynamics provided a new methodology for the correct evaluation of the stability of rotor systems.

3. Finding Non Dimensional Parameters

To analyse the parameters which are influencing the bearing characteristics it is very convenient to apply Buckingham π- Method. Because there is nearly more than 20 parameters are to be used. In other methods, it is difficult task to compute all the dimensionless numbers. Before applying Buckingham π-theorem, first the parameters of bearing which are to be considered for analysing the characteristics of bearing are as follows:

Table- 1: List of Bearing Parameters

S.No	Parameters	L	M	T	Θ
1	Reaction Force(Fr)	1	1	-2	0
2	Acceleration due to gravity(g)	1	0	-2	0
3	Shaft speed(n)	0	0	-1	0
4	Power (p)	2	1	-3	0
5	Heat Generated in the bearing(q)	2	1	-2	0
6	Surface temperature of the bearing(T_a)	0	0	0	1
7	Load carrying Capacity(N)	1	1	-2	0
8	Frequency of the bearing(F_b)	0	0	-1	0
9	Dynamic viscosity(μ)	-1	1	-1	0
10	Bearing Size(D_{ball})	1	0	0	0
11	Clearance of Ball Bearing (C_{ball})	1	0	0	0
12	Thermal conductivity(k)	1	1	-3	-1
13	Convective heat transfer coefficient(h)	0	1	-3	-1
14	Shaft density (ρ)	-3	1	0	0
15	Flexural Rigidity(EJ)	1	3	-2	0
16	Moment of Inertia(I)	4	0	0	0
17	Stiffness(s)	0	1	-2	0
18	Thermal conductivity(k_{ref})	1	1	-3	-1
19	Shaft speed(n ref)	0	0	-1	0
20	Load carrying Capacity(N_{ref})	1	1	-2	0
21	Frequency of the bearing (Fb_{ref})	0	0	-1	0
22	Bearing Size($D_{ball\ ref}$)	1	0	0	0
23	Dynamic viscosity(μ_{ref})	-1	1	-1	0
24	Temperature initial($T_{initial}$)	0	0	0	1

4. Evaluating π-terms

The number of bearing parameters considered as per Table-1,the π-terms are evaluated by considering four repeating variables which are Heat transfer coefficient(h), Heat Generated in the bearing(q), Thermal conductivity(k), Power (p),therefore the number of π-terms is 20.The 20 π-terms are represented in terms of equations are given below.The equations are solved by Buckingham pi-theorem and the equations (1-20) are in matrix form is given in equation (21).The matrix $B_1$is the solution matrix and matrix A is original dimensional matrix and in which A_S is a sub matrix of decisive quantities and A_Z is the residual sub matrix. The solution matrix(B_1) in terms of equations are given below (equations 22-41).

$$\Pi_1 = h^{a1}q^{b1}k^{c1}p^{d1}.Fr \tag{1}$$
$$\Pi_2 = h^{a2}q^{b2}k^{c2}p^{d2}.g \tag{2}$$
$$\Pi_3 = h^{a3}q^{b3}k^{c3}p^{d3}.n \tag{3}$$
$$\Pi_4 = h^{a4}q^{b4}k^{c4}p^{d4}.Ta \tag{4}$$
$$\Pi_5 = h^{a5}q^{b5}k^{c5}p^{d5}.W \tag{5}$$
$$\Pi_6 = h^{a6}q^{b6}k^{c6}p^{d6}.fb \tag{6}$$
$$\Pi_7 = h^{a7}q^{b7}k^{c7}p^{d7}.\mu \tag{7}$$
$$\Pi_8 = h^{a8}q^{b8}k^{c8}p^{d8}.Dball \tag{8}$$

$$\Pi_9 = h^{a9}q^{b9}k^{c9}p^{d9}.T_{initial} \quad (9)$$
$$\Pi_{10} = h^{a10}q^{b10}k^{c10}p^{d10}.fb_{\ ref} \quad (10)$$
$$\Pi_{11} = h^{a11}q^{b11}k^{c11}p^{d11}.\mu_{\ ref} \quad (11)$$
$$\Pi_{12} = h^{a12}q^{b12}k^{c12}p^{d12}.Dball_{\ ref} \quad (12)$$
$$\Pi_{13} = h^{a13}q^{b13}k^{c13}p^{d13}.k_{\ ref} \quad (13)$$
$$\Pi_{14} = h^{a14}q^{b14}k^{c14}p^{d14}.n_{\ ref} \quad (14)$$
$$\Pi_{15} = h^{a15}q^{b15}k^{c15}p^{d15}.N_{\ ref} \quad (15)$$
$$\Pi_{16} = h^{a16}q^{b16}k^{c16}p^{d16}.\rho \quad (16)$$
$$\Pi_{17} = h^{a17}q^{b17}k^{c17}p^{d17}.E_J \quad (17)$$
$$\Pi_{18} = h^{a18}q^{b18}k^{c18}p^{d18}.I \quad (18)$$
$$\Pi_{19} = h^{a19}q^{b19}k^{c19}p^{d19}.S \quad (19)$$
$$\Pi_{20} = h^{a20}q^{b20}k^{c20}p^{d20}.C_{\ ball} \quad (20)$$

$$B_1= -[(A_Z)^T)^{-1}*(B_Z)^T] \quad (21)$$

After solving, the π-terms are as stated below

$$\Pi_1 = h^{-1}q^{-1}k^1p^0.F_r \quad (22)$$
$$\Pi_2 = h^1q^2k^{-1}p^{-2}.g \quad (23)$$
$$\Pi_3 = h^0q^1k^0p^{-1}.n \quad (24)$$
$$\Pi_4=h^{-1}q^0k^2p^{-1}.T_a \quad (25)$$
$$\Pi_5 = h^{-1}q^{-1}k^1p^0.W \quad (26)$$
$$\Pi_6= h^0q^1k^0p^{-1}.f_b \quad (27)$$
$$\Pi_7= h^{-3}q^{-2}k^3p^1.\mu \quad (28)$$
$$\Pi_8 = h^1q^0k^{-1}p^0.D_{ball} \quad (29)$$
$$\Pi_9= h^{-1}q^0k^2p^{-1}.T_{initial} \quad (30)$$
$$\Pi_{10}= h^0q^1k^0p^{-1}.fb_{\ ref} \quad (31)$$
$$\Pi_{11}= h^{-3}q^{-2}k^3p^1.\mu_{\ ref} \quad (32)$$
$$\Pi_{12}= h^1q^0k^{-1}p^0.D_{ball\ ref} \quad (33)$$
$$\Pi_{13}= h^0q^0k^{-1}p^0.k_{\ ref} \quad (34)$$
$$\Pi_{14}= h^0q^1k^0p^{-1}.n_{\ ref} \quad (35)$$
$$\Pi_{15}= h^{-1}q^{-1}k^1p^0.N_{\ ref} \quad (36)$$
$$\Pi_{16} = h^{-5}q^{-3}k^5p^2.\rho \quad (37)$$
$$\Pi_{17}= h^{-5}q^{-7}k^5p^4.EJ \quad (38)$$
$$\Pi_{18}= h^4q^0k^{-4}p^0.I \quad (39)$$
$$\Pi_{19}= h^{-2}q^{-1}k^2p^0.S \quad (40)$$
$$\Pi_{20}= h^1q^0k^{-1}p^0.C_{\ ball} \quad (41)$$

5. Result and discussions

Among all these equations $\pi_4$and π_9, $\pi_6$and π_{10}, $\pi_7$and π_{11}, $\pi_8$and π_{12}, $\pi_3$andπ_{14} terms are same. Therefore only 13 π terms are influencing the characteristics of bearing. In this analysis the 13 non-dimensional parameters are considered based on the temperature variance. For air the convective heat transfer coefficient is varying according to temperature so each non dimensional parameter is to be studied as per different convective heat transfer coefficient of air ranging from 5 to 25 W/m^2 K.The above solved π terms are investigated theoretically by considering variation of convective heat transfer of air is one of the parameter and varying non dimensional L/D ratios.The graphs plotted for different composed criteria's by varying the L/D ratio and convective heat transfer coefficient are shown in Fig. 1(a) & 1(b). The plots show that there is significant effect influence of Convective heat transfer coefficient. This is observed for all composed criteria`s. For the plot between π_6 and L/D ratio the influence of power consumption is clearly observed. For the plots between $\pi_{7\text{-}11}$has no influence on bearing system.

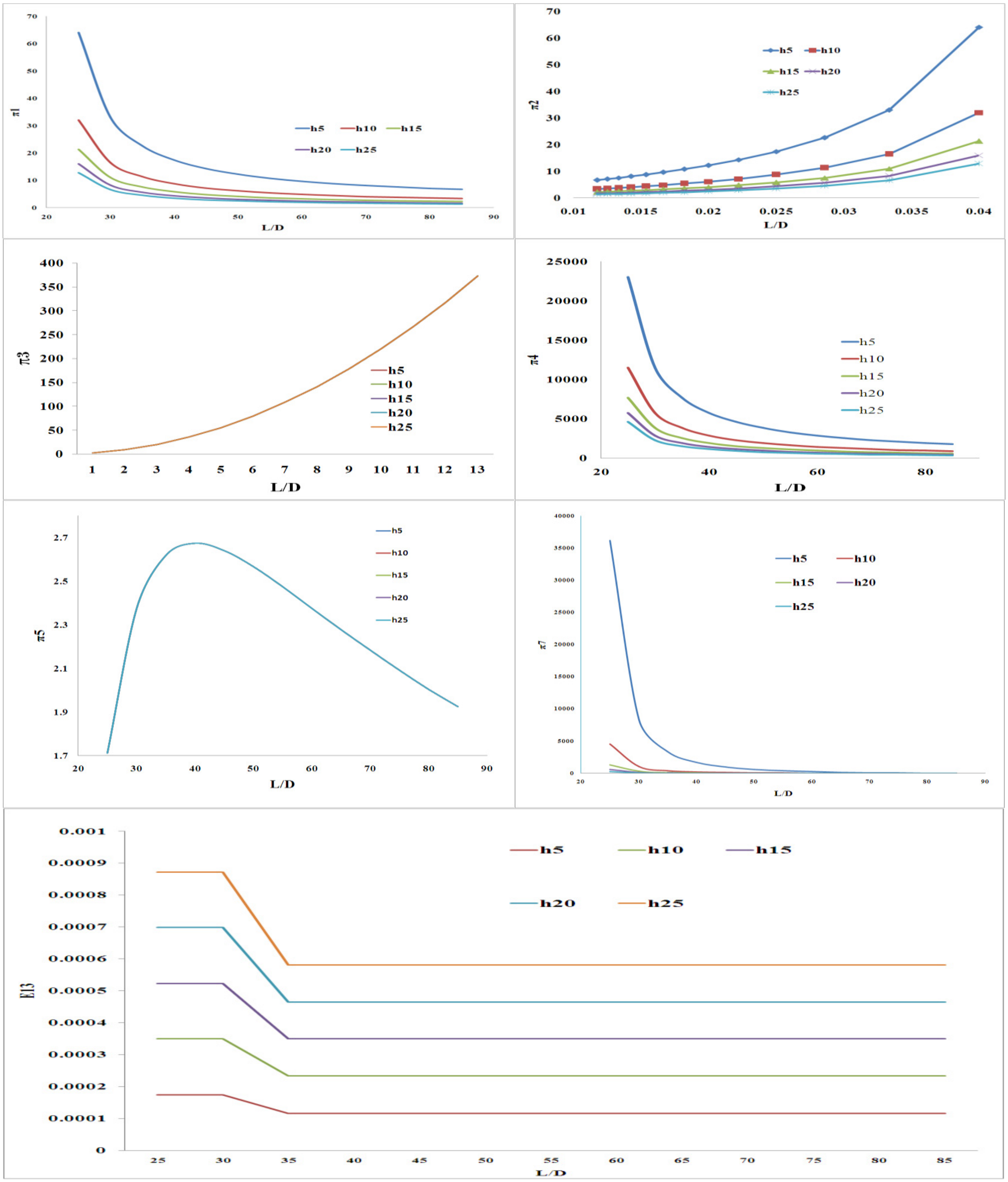

Fig.1(a) shows plot between πvalues and L/D ratios at different h values

Conclusions

The presented paperis to analyse the bearing parameters under the influence of convective heat transfer coefficient. This analysis is done by application of Buckingham π-theorem. From this method 20 dimensionless quantities are obtained and each dimensionless quantity was studied by plotting the graphs between L/D ratio and different π-terms at different values of convective heat transfer coefficients ranging from 5 to 25 W/m^2K. Use of regression Dimensional Analysis focuses on fewer terms than conventional regression. The models generated are guaranteed to be dimensionally homogeneous. Further, the constants determined in fitting the models are, unlike in standard regression analysis, true dimensionless constants, unaffected by changes in units of measurement. This method can be used to find the influence of bearing under variation of very low convective heat transfer coefficients. The application can be adopted for bearings running at very low temperatures. The present work is only on theoretical analysis of the different bearing parameters and it is to be checked by conducting the experiment on bearing system. The dimensional analysis method reveals how an equation describing a physical system can be reduced to a function of set of dimensionless products which are usually fewer than the prescribed set of explanatory variables. This may even reduce the problem to simply determining a constant in exceptional cases.

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